PROPOSED STANDARD SOLAR-RADIATION CURVES FOR ENGINEERING USE.

BY

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Questions regarding sunlight are constantly occurring in illuminating engineering: How does illumination vary with the season and with the time of day? How much solar radiant energy will be absorbed by a given material to be used in the tropics? How effective are various infrared-absorbing glasses? What is the relation between altitude and amount of ultra-violet radiation? How does the color of sunlight vary with latitude? A great number of data have been collected on such subjects, but little attempt seems to have been made to compare these data, to correlate them, and to put them in a convenient form for engineering use.

In this paper a comparison is made of data obtained by various investigators on the spectral distribution of sunlight outside the earth's atmosphere. On the basis of this comparison, a *proposed standard curve*, expressed in absolute units, is recommended for use in engineering computations dealing with sunlight. Data for atmospheric transmission are given also, so that the resulting spectral irradiation curve can be computed for any part of the earth's surface at any time and for any elevation above sea level.

Since a large proportion of the world's population lives not far above sea level, an elevation near zero is the most generally useful one. Accordingly a set of *proposed standard curves* for the spectral distribution of direct sunlight has been computed for sea level. With such curves available, it is a simple matter to compute the illumination, the heating effect, the color, etc. A number of such calculations are carried through to check the proposed standard curves against data obtained in quite different ways. These checks indicate that the method yields results of sufficient accuracy for engineering use.

2. THE SOLAR CONSTANT.

The radiant energy at normal incidence, received at the surface of the earth from the sun, is subject to wide variations because of the following factors:

1. Variations in the sun itself.
2. Variations in the distance from earth to sun.
3. Differences in atmospheric scattering
   a. By air molecules
   b. By water vapor
   c. By dust
4. Differences in atmospheric absorption by O_2, O_3, H_2O, CO_2, etc.

In 1881, Langley applied his newly developed bolometer to the measurement of spectral irradiation from the sun.[1] Measurements made with the sun at various zenith angles allowed him to compensate for the effect of the atmosphere and thus to calculate the solar irradiation,[2] at normal incidence, outside the atmosphere at the mean solar distance. This quantity is called the *solar constant.* Since 1902 the Smithsonian Institution has been engaged in a painstaking study of the solar constant and has collected evidence [3] that this quantity actually varies with time, the maximum fluctuations being approximately $\pm$ 1.5 per cent. The mean value of the Smithsonian results [3] from 1920 to 1934 is 1.9408 cal. cm.$^{-2}$ min.$^{-1}$. However, this value is on the basis of the 1913 Smithsonian standard,[4] which is now believed to be 2.3 per cent. high.[5] Thus the best value for the mean solar constant appears to be

$$1.9408 \times 0.977 = 1.896 \text{ cal. cm.}^{-2} \text{ min.}^{-1}.$$

On the basis of 4.1835 joules = 1 gram-calorie, the solar

[1] S. P. Langley, "The Bolometer and Radiant Energy," Am. Acad., *Proc.*, **8**, 1881, p. 342.

[2] The nomenclature used in this paper is in accordance with Moon, "Scientific Basis of Illuminating Engineering," New York, 1936, p. 560.

[3] C. G. Abbot, "Solar Radiation and Weather Studies," Smithsonian Misc. Coll., **94**, No. 10, 1935, p. 12.

[4] Smithsonian Institution, *Annals of Astrop. Obs.*, 3, 1913, p. 134.

[5] Abbot and Aldrich, "The Scale of Solar Radiation," Smithsonian Misc. Coll., **92**, No. 13, 1934.

constant is therefore

$$1322 \text{ watt m.}^{-2}.$$

This value is in good agreement with independent determinations.[6]

The actual irradiation at normal incidence may differ from the foregoing value, not only because of the effect of the atmosphere and because of fluctuations in the sun itself, but also because of variations in solar distance. This factor produces a maximum change in irradiation of approximately $\pm$ 3.5 per cent., the exact amount at any time of year being determined by well-known methods which need not be considered here.

3. SPECTRAL IRRADIATION CURVE.

As the next step, consider the curve of spectral irradiation outside the atmosphere. By far the greatest amount of information on this subject has been obtained by the Smithsonian Institution,[8] and some of their results are plotted in Fig. 1. Data obtained at Potsdam by Wilsing [7] are also included, as are the results of Pettit [9] and of Fabry and Buisson.[10]

The data of Fig. 1 are in arbitrary units, and thus it is permissible to move any set of data up and down on the diagram. The Smithsonian results were plotted directly from the table given by Abbot, Fowle and Aldrich,[8] while the other sets of points were shifted vertically to give the best match. The blackbody curve [11] was adjusted so that its

[6] See, for instance, C. Tingwaldt, "Ein neues Pyrheliometer für Absolutmessungen," *Zs. f. Instrumentenkde.*, **51**, 1931, p. 593. A. Unsöld, "Physik der Sternatmosphären," Berlin, 1938, p. 27.

[7] I. Wilsing, "Über die Helligkeitsverteilung im Sonnenspektrum nach bolometrischen Messungen," Astrop. Obs. Potsdam, *Publikationen*, **32**, No. 72, 1917, pp. 91, 92.

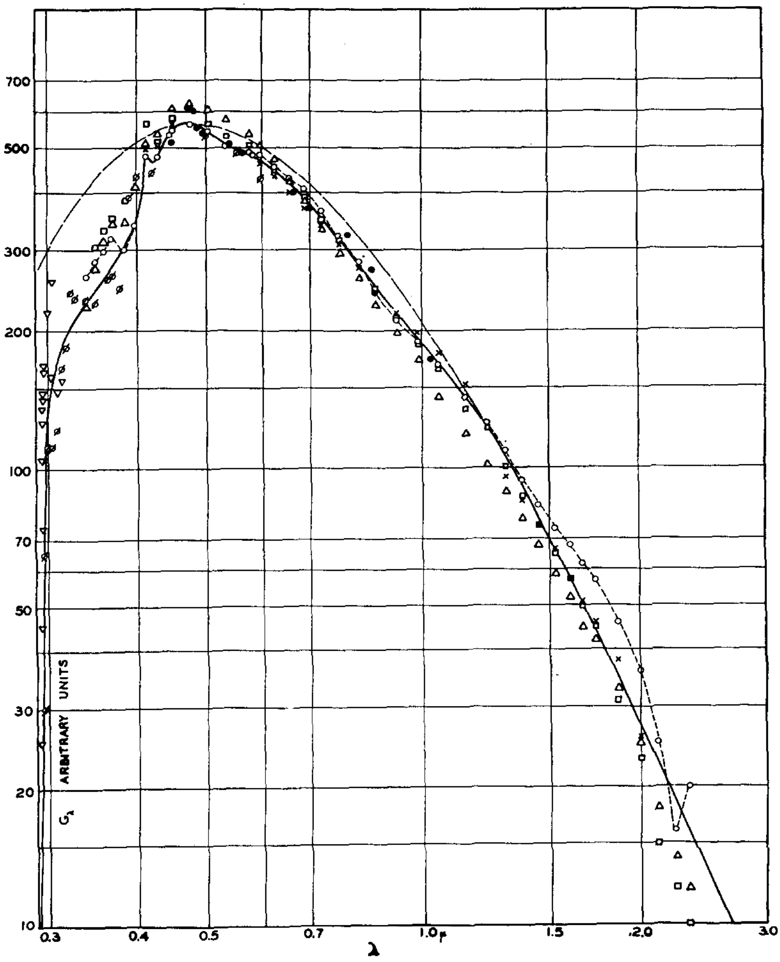
[8] Abbot, Fowle, and Aldrich, "The Distribution of Energy in the Spectra of the Sun and Stars," Smithsonian Misc. Coll., **74**, No. 7, 1923, p. 15.

[9] E. Pettit, "Measurements of Ultraviolet Solar Radiation," *Astrop. J.*, **75**, 1932, p. 185.

[10] Fabry and Buisson, "A Study of the Ultraviolet End of the Solar Spectrum," *Astrop. J.*, **54**, 1921, p. 297.

[11] P. Moon, "Tables of Planck's Function from 3500 to 8000° K.," *J. Math. and Phys.*, **16**, 1937, p. 133.

FIG. 1.



Solar irradiation on a surface outside the atmosphere.

- ▽ Fabry & Buisson
- φ Pettit
- ● Wilsing
- △ 1903–1910 (Smithsonian Institution)
- × 1903–1910 (omitting quartz results) (Smithsonian Institution)
- □ 1916–1918 (Smithsonian Institution)
- ○ - - ○ 1920–1922 (Smithsonian Institution)
- — — Blackbody, 6000° K.
- ——— Proposed standard curve

maximum was approximately equal to the maximum of the 1920–22 curve.

The best result of the Smithsonian Institution is usually believed to be [8] the weighted mean of the 1920 and 1922 results, which is shown by the circles and the dotted curve of Fig. 1. In common with all other data, this curve indicates a narrow peak at 0.48 μ and a depression at 0.55 μ. However, in the region from 0.60 to 0.75 μ, the curve is higher than any of the other data, while in the region 0.85 to 1.0 μ it sinks below most of the other data. Thus there seems to be a question as to whether the 1920–22 representation is the best one in these regions. In an attempt to get the smoothest curve that is in agreement with the available information, I have used the 1920–22 results from 0.40 to 0.55 μ but have departed slightly from the Smithsonian results at longer wavelengths, as indicated by the heavy curve of Fig. 1. At wavelengths shorter than 0.40 μ, there is reason to believe that the Smithsonian results are high because of scattered light in the monochromator. I have therefore favored the Pettit values between 0.32 and 0.40 μ and have used the Pettit and the Fabry-Buisson values at still shorter wave-lengths. In both of these investigations stray light was eliminated.

All the data indicate that the broad peak associated with the blackbody curve is not obtained in the solar spectrum. The depression of the solar curve from 0.50 to 1.0 μ appears to be so well substantiated that there would be no justification in substituting the Plankian curve in this region. At longer wave-lengths, however, the blackbody curve is actually a much better approximation to all the data than is the 1920–22 curve. Thus I have employed the 6000° K. Planckian curve for all wave-lengths beyond 1.25 μ. Radiation at wave-lengths beyond 2.3 μ is strongly absorbed by H_2O and CO_2 and is so weak in any case, that no satisfactory measurements are available.[12] Abbot concludes [13] from existing data that the radiant energy for $\infty > \lambda > 2.5$ is almost exactly 2 per cent. of the total. Because of this low value, the exact form of curve assumed beyond 2.5 μ will have little influence on the

[12] See, however, Smithsonian Institution, *Annals of Astrop. Obs.*, **5**, 1932, p. 104.

[13] Ibid., **5**, 1932, p. 105.

results and the Planckian form appears to be the simplest representation.

It is interesting to note that the maximum spread of the experimental points in the visible and near infrared is generally not more than 10 per cent., while the solid and the dotted curves differ by 5 per cent. or less. At wave-lengths longer than 1.0 μ and at wave-lengths shorter than 0.4 μ, the scattering of points is much greater and differences of as much as 50 per cent. are found.

Consideration of all data led to the tentative adoption of the heavy curve shown in Fig. 1. The integral,

$$G = 1322 = k \int_0^\infty G_{\lambda\,\mathrm{rel}} d\lambda, \tag{1}$$

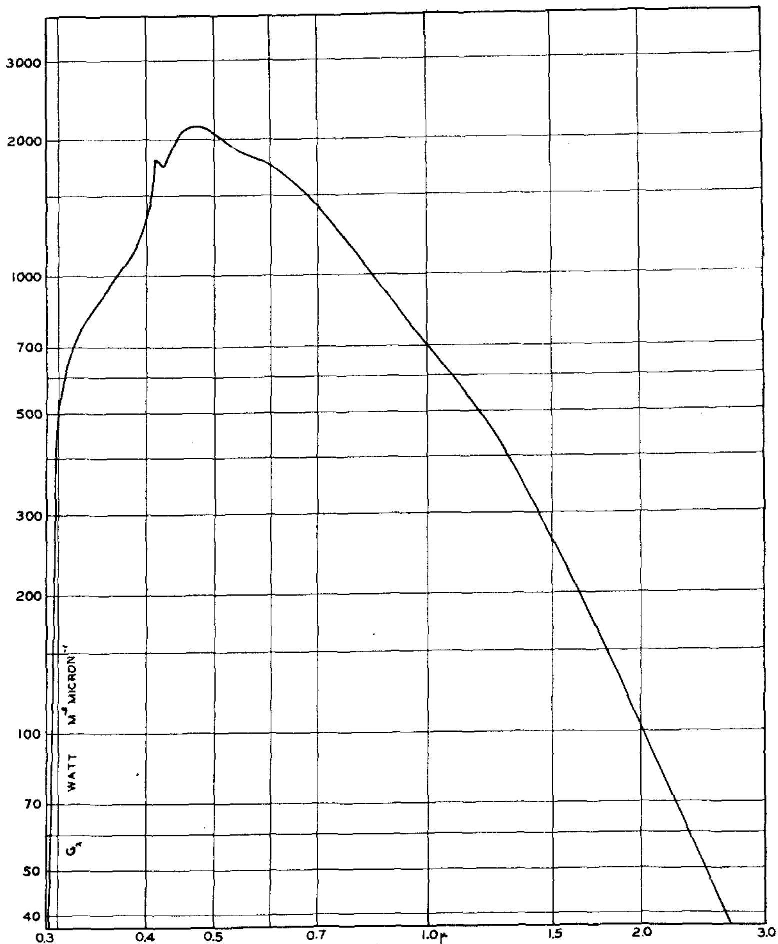
for this curve was evaluated mechanically. In this way, the scale factor k was found to be 3.756. Multiplication of the ordinates of the heavy curve of Fig. 1 by this value of k gave the actual irradiation G in watt m.$^{-2}$ per micron. These values are given in Fig. 2 and Table I and are proposed as a standard for use in engineering computations dealing with solar irradiation.

For the sake of definiteness, I have brought the curve down to zero at 0.290 μ. The actual solar curve outside our atmosphere is not zero at this wave-length, though there is evidence that the actual irradiation is very low. Experimental difficulties are great, however, since atmospheric ozone causes almost complete absorption at 0.290 so that it is difficult if not impossible to determine how much of the absorption occurs in the earth's atmosphere and how much in the sun. This distinction may worry the astrophysicist; but it should cause no trouble in the present paper, where the curve is used merely as a convenient norm from which irradiation of the earth's surface is obtained.

4. ATMOSPHERIC SCATTERING.

The direct radiation from the sun is attenuated by the scattering effect of molecules of nitrogen, oxygen, and other constituents of the atmosphere. Since the molecular size is very small compared with the wave-lengths under consideration, the Rayleigh fourth-power relation should apply. That it does apply was shown by Fowle,[14] who was able to obtain a fairly precise determination of Loschmidt's number by sub-

FIG. 2.



Proposed standard solar irradiation curve. Watt m.$^{-2}$ micron^{-1} on a surface perpendicular to the sun's rays, outside the atmosphere and at the mean solar distance.

stituting atmospheric transmission data obtained on Mount Wilson in Rayleigh's formula.

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[14] F. E. Fowle, "The Atmospheric Scattering of Light," Smithsonian Misc. Coll., 69, No. 3, 1918.

TABLE I.

Solar Irradiation at Normal Incidence, Outside Atmosphere, and at Mean Solar Distance.

Solar constant = 1322 watt m.$^{-2}$.

λ (microns)	G_λ (watt m^{-2} μ^{-1})	λ (microns)	G_λ (watt m^{-2} μ^{-1})	λ (microns)	G_λ (watt m^{-2} μ^{-1})
0.290	0	0.400	1304	1.0	706
0.291	190	0.405	1427	1.1	590
0.292	250	0.410	1728	1.2	488
0.293	289	0.413	1803	1.3	395
0.294	318	0.420	1766	1.4	319
		0.424	1742		
0.295	345	0.430	1788	1.5	260
0.296	368	0.440	1939	1.6	214
0.297	389			1.7	177
0.298	410	0.45	2036	1.8	148
0.299	430	0.46	2096	1.9	124
		0.47	2119		
0.300	450	0.48	2127	2.0	105
0.301	470	0.49	2103	2.1	89.3
0.302	489			2.2	76.4
0.303	507	0.50	2061	2.3	65.8
0.304	524	0.51	2000	2.4	56.9
		0.52	1954		
0.305	540	0.53	1912	2.5	49.5
0.306	556	0.54	1894	2.6	43.2
0.307	571			2.7	37.9
0.308	586	0.55	1878	2.8	33.4
0.309	601	0.56	1861	2.9	29.5
		0.57	1841		
0.310	616	0.58	1819	3.0	26.1
0.311	628	0.59	1795	3.1	23.3
0.312	640			3.2	20.8
0.313	652	0.60	1762	3.3	18.6
0.314	664	0.61	1727	3.4	16.6
		0.62	1690		
0.315	676	0.63	1653	3.5	14.9
0.316	686	0.64	1616	3.6	13.5
0.317	696			3.7	12.2
0.318	706	0.65	1579	3.8	11.1
0.319	716	0.66	1543	3.9	10.1
		0.67	1508		
0.320	726	0.68	1473	4.0	9.20
0.325	762	0.69	1439	4.1	8.44
0.330	796			4.2	7.75
0.335	826	0.70	1405	4.3	7.10
0.340	856	0.71	1371	4.4	6.50
0.345	886	0.72	1337		
		0.73	1304	4.5	5.93
0.350	916	0.74	1270	4.6	5.44
0.360	976			4.7	4.99
0.370	1046	0.75	1236	4.8	4.62
0.380	1121	0.80	1097	4.9	4.32
0.390	1202	0.85	976		
		0.90	871	5.0	4.06
		0.95	781		

The study of a large number of data allowed Fowle [15] to separate the scattering effect of water vapor from true atmospheric scattering. The transmission factor of the atmosphere directly above Mount Wilson and with no water vapor may be written

$$\tau_{a\lambda} = 10^{-k_{a\lambda}}, \tag{2}$$

where $k_{a\lambda}$ is a function of wave-length. The values of $k_{a\lambda}$, obtained from the experimental values of $\tau_{a\lambda}$, are plotted against wave-length in Fig. 3. Fowle found that between 0.35 and 0.50 μ, the values plotted as a straight line having a slope of -4, as would be expected from Rayleigh's theory. At wave-lengths from 0.5 to 0.7 μ, the points depart from the straight line because of the Chappuis ozone band, while at wave-lengths beyond 1 μ the transmission factor is so nearly unity that experimental errors make the points completely unreliable. There is every reason to believe, however, that the Rayleigh relation continues to apply at all wave-lengths in the solar spectrum, so that the transmission factor associated with atmospheric scattering may be written

$$\tau_{a\lambda} = 10^{-0.00380/\lambda^4}, \tag{3}$$

where λ is in microns. Eq. (3) applies when the sun is at the zenith and the barometric pressure is 760 mm.

I have also plotted (Fig. 3) Fowle's values for the scattering effect of water vapor. In this case the curve has a slope of -2 instead of -4; and the associated transmission factor may be written

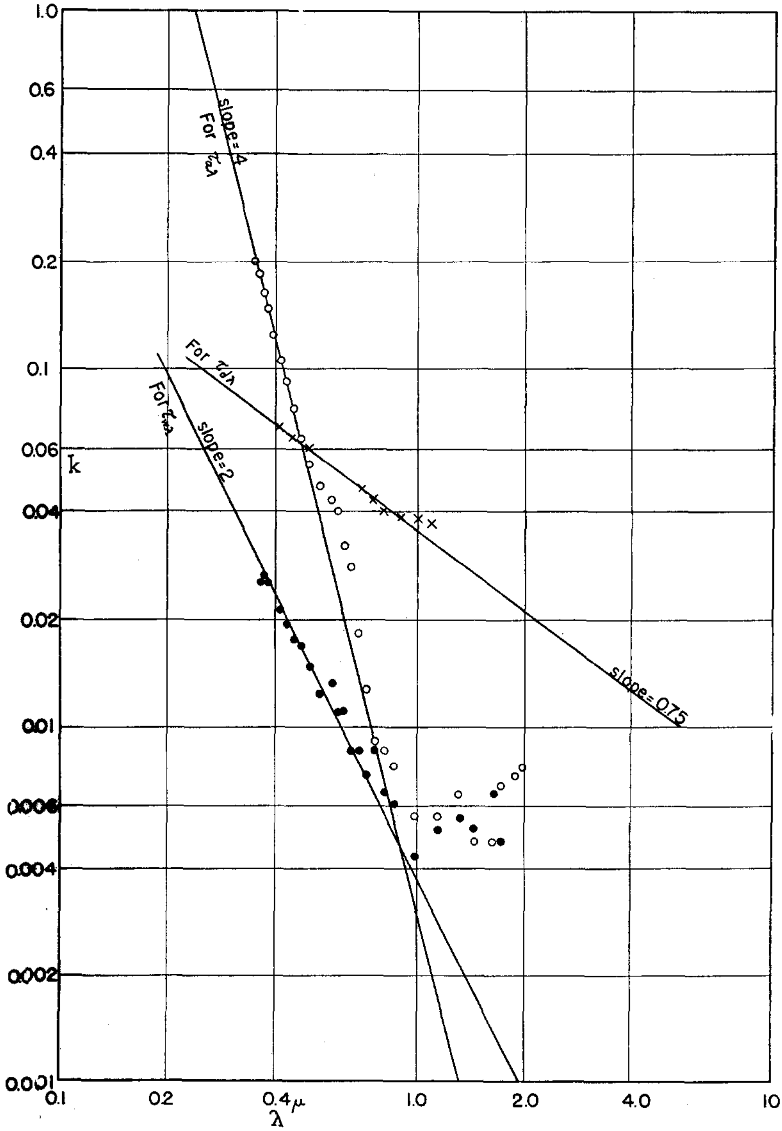
$$\tau_{w\lambda} = 10^{-0.0075/\lambda^2}. \tag{4}$$

for zenith sun and with 20 mm. of precipitable water above the station.

At an elevation of 1000 m. or more, these two transmission factors are usually sufficient to account for all the observable scattering of light. When dust is present, however, as it always is near sea level, it will introduce additional scattering. The particles are rather large (order of 1 μ diameter) so that the effect of wave-length is less marked than with molecular scattering, and it is customary to assume that scattering by

[15] F. E. Fowle, "Avogadro's Constant and Atmospheric Transparency," *Astrop. J.*, 40, 1914, p. 435.

FIG. 3.



Atmospheric scattering. Values of k in the equation:

$\tau_\lambda = 10^{-k}$.

dust is independent of λ. A more thorough analysis is shown in Fig. 3. Eqs. (3) and (4) were used in calculating the total transmission factor τ_λ at sea level and for 20 mm. of precipitable water. The resulting curve was compared with the mean curve obtained at Washington, D. C.[16] The ratio between the ordinates of the two curves was attributed to dust, and the corresponding values of $k_{d\lambda}$ were plotted in Fig. 3. The slope is seen to be -0.75 and the accompanying transmission factor is

$$\tau_{d\lambda} = (10)^{-0.0353/\lambda^{0.75}}. \tag{5}$$

According to Hand,[17] the average number of dust particles at Washington is approximately 800 per cubic cm.; so for a first approximation one can associate Eq. (5) with 800 particles/cm.^{3} and can calculate $\tau_{d\lambda}$ for other amounts of dust by proportionate changes in the exponent.

The total effect of scattering is, by the Bouguer relation,

$$\tau_\lambda = \left[(\tau_{a\lambda})^{p/760}(\tau_{w\lambda})^{w/20}(\tau_{d\lambda})^{d/800}\right]^m, \tag{6}$$

where $\tau_{a\lambda}$ = spectral transmission factor associated with scattering by a dry atmosphere at 760 mm. pressure and with sun at zenith,

$\tau_{w\lambda}$ = spectral transmission factor associated with scattering by water vapor, for 20 mm. of precipitable water directly above the observer,

$\tau_{d\lambda}$ = spectral transmission factor associated with dust, for 800 particles/cm^3 at the level of the observer, and with sun at zenith,

p = barometric pressure (mm.),

w = depth of precipitable water (mm.),

m = sec θ = air mass ($m = 1$ for sun at zenith),

θ = zenith angle of sun.

A convenient way of obtaining τ_λ from Eq. (6) is to plot the three separate factors on logarithmic paper. A pair of dividers can then be used to obtain the sum of the three distances below the line $\tau = 1$, giving the curve of τ_λ vs. λ, which can be plotted very rapidly in this manner.

[16] Smithsonian Institution, *Annals of Astrop. Obs.*, **2**, 1908, p. 113.

[17] I. F. Hand, Review of United States Weather Bureau Solar Radiation Investigations, *Mon. Weather Rev.*, **65**, 1937, Table 12, p. 441.

5. OZONE.

The measured transmission curves for the atmosphere contain also the effect of *absorption* of radiant energy by ozone. The transmission factor associated with the Chappuis band (0.5 to 0.7 μ) was taken from Wulf,[18] who made these measurements in the solar spectrum with a layer of ozone equivalent to 3.8 mm. at normal temperature and pressure. The results (smoothed) are given in Table II.

At shorter wave-lengths, the Hartley band and the Huggins band extend to approximately 0.35 μ. These data (Table II) are based on the research of Läuchli.[19] It has not been settled definitely if appreciable error is introduced by applying the Bouguer formula to these data in finding the effect of a change in the amount of the ozone. The formula has been used by many investigators,[19,20] however, and with good results. It will be used in the present paper.

6. COMPARISON OF COMPUTED AND MEASURED TRANSMISSION CURVES.

The heavy curve of Fig. 4 was computed by use of the formulas and data of the preceding sections and for a barometric pressure of 760 mm., 20 mm. of precipitable water, 800 dust particles/cm.^{3}, and 3.8 mm. of ozone. The Smithsonian results for Washington [16] are in good agreement except at wave-lengths below 0.4 μ, where they are probably in error because of stray light in the monochromator. The two solid circles are measurements of Fabry and Buisson [10] at Marseille, France. Since this is also an industrial city only slightly above sea level, the results should be comparable with those obtained at Washington.

A more complete comparison between the theoretical curve and the results of Fabry and Buisson is shown in Fig. 5. The experimental results are slightly above the curve, which

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[18] O. R. Wulf, "The Determination of Ozone by Spectrobolometric Measurements," Smithsonian Misc. Coll., **85**, No. 9, 1931, p. 9.

[19] A. Läuchli, "Zur Absorption der ultravioletten Strahlung im Ozon," *Zs. f. Phys.*, **53**, 1929, p. 92.

[20] Fabry and Buisson, "L'absorption de l'ultra-violet par l'ozone et la limite du spectre solaire," *J. de Phys.*, **3**, 1913, p. 196. F. E. Fowle, "Atmospheric Ozone," Smithsonian Misc. Coll., **81**, No. 11, 1929. R. W. Ladenburg, "Light Absorption and Distribution of Atmospheric Ozone," *J. O. S. A.*, **25**, 1935, p. 259.

TABLE II.

*Transmission Factor for Ozone.**
(3.8 mm. layer at 760 mm. pressure and 0° C.)

λ (microns)	α	τ_λ	λ	α	τ_λ
0.290	1.65	5.38×10^{-7}	0.350	0.0008	1.000
0.291	1.46	2.82×10^{-6}			
0.292	1.29	1.26×10^{-5}	0.48		1.000
0.293	1.11	6.03×10^{-5}	0.49		0.999
0.294	1.00	1.59×10^{-4}			
0.295	0.89	4.17×10^{-4}	0.50		0.996
0.296	0.79	0.00100	0.51		0.993
0.297	0.70	0.00219	0.52		0.989
0.298	0.61	0.00480	0.53		0.984
0.299	0.52	0.0105	0.54		0.979
0.300	0.460	0.0178	0.55		0.973
0.301	0.405	0.0288	0.56		0.965
0.302	0.360	0.0427	0.57		0.960
0.303	0.315	0.0631	0.58		0.955
0.304	0.275	0.0890	0.59		0.950
0.305	0.241	0.122	0.60		0.955
0.306	0.213	0.155	0.61		0.960
0.307	0.190	0.190	0.62		0.965
0.308	0.164	0.238	0.63		0.973
0.309	0.144	0.283	0.64		0.979
0.310	0.126	0.332	0.65		0.984
0.311	0.112	0.376	0.66		0.989
0.312	0.099	0.421	0.67		0.993
0.313	0.087	0.467	0.68		0.996
0.314	0.076	0.515	0.69		0.999
0.315	0.066	0.561	0.70		1.000
0.316	0.057	0.607			
0.317	0.050	0.645			
0.318	0.044	0.680			
0.319	0.039	0.711			
0.320	0.0345	0.740			
0.325	0.0180	0.855			
0.330	0.0092	0.922			
0.335	0.0048	0.960			
0.340	0.0025	0.980			
0.345	0.0013	0.990			

*From data of Läuchli, Zs. f. Phys., 53, 1929, p. 92, and Wulf, Sm. Misc. Coll., 85, No. 9, 1931, p. 9. The transmission factor is

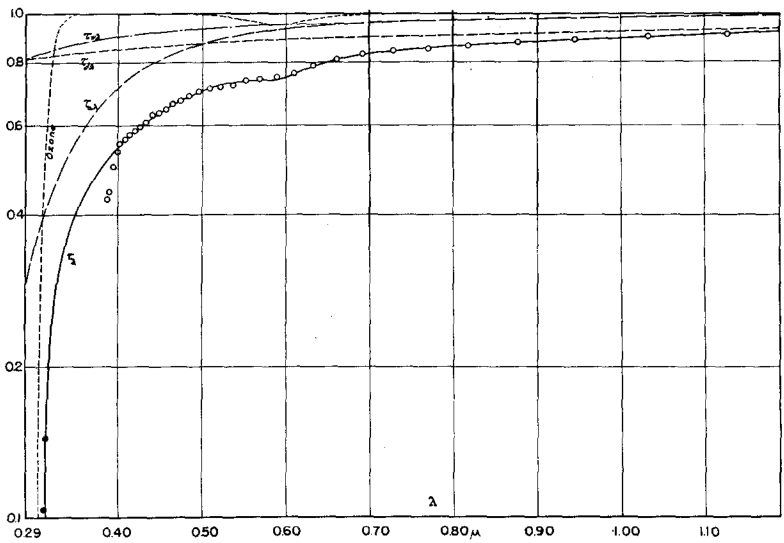
$$\tau_\lambda = (10)^{-\alpha x}$$

where x is the thickness (NTP) in mm.

probably indicates a smaller amount of ozone than the 3.8 mm. used in the calculations.

Another check for the calculated curve in the ultraviolet is obtained by using the experimental results of Pettit.[9] These measurements were made at Tucson, Arizona (760 m.) in May 1931. Pettit gives the ozone as 1.8 mm. but does not state the amount of water or dust. An assumption of 10 mm.

FIG. 4.



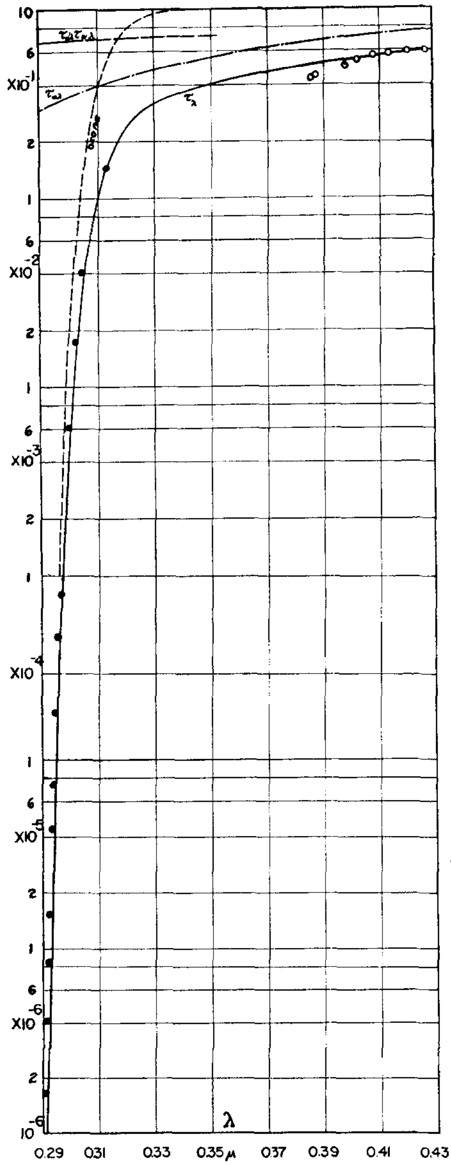
Atmospheric transmission factor for zenith sun.
○ Experimental (Abbot)
——— Calculated for p = 760 mm., w = 20 mm., d = 800, ozone = 3.8 mm.

H_2O and no dust gives the heavy curve of Fig. 6. The points are in good agreement except at the shortest wave-lengths.

Another case is shown in Fig. 7, where the points represent mean values obtained on Mount Whitney, California[21] (4420 m.). At this elevation there is no dust and practically no water vapor. The mean value of 0.8 mm. of precipitable water[22] has almost negligible effect and was omitted from the calculations. The points are seen to hug the calculated curve of $\tau_{a\lambda}$ except in the ozone bands. Other data, obtained in

[21] *Smithsonian Astrop. Obs.*, 3, 1913, p. 136.
[22] Ibid., p. 113.

FIG. 5.



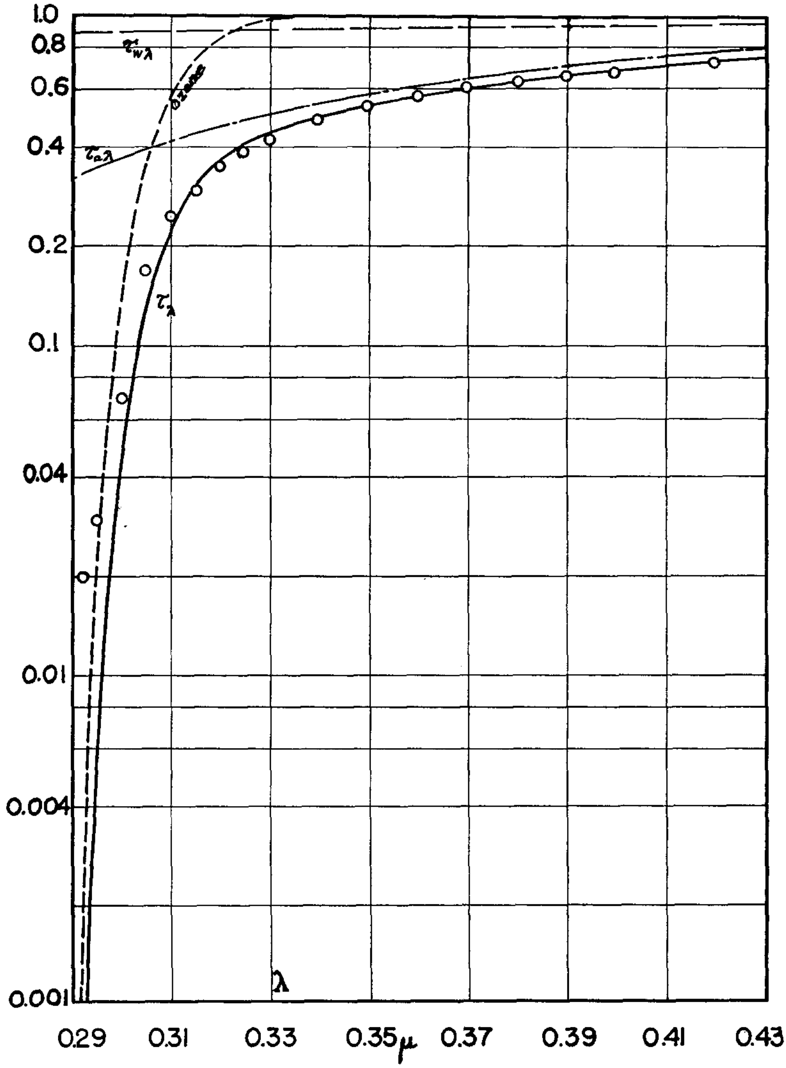
Atmospheric transmission factor for zenith sun.

○ Experimental, Abbot

● Experimental, Fabry & Buisson

——— Calculated for p = 760 mm., w = 20 mm., d = 800, ozone = 3.8 mm.

FIG. 6.



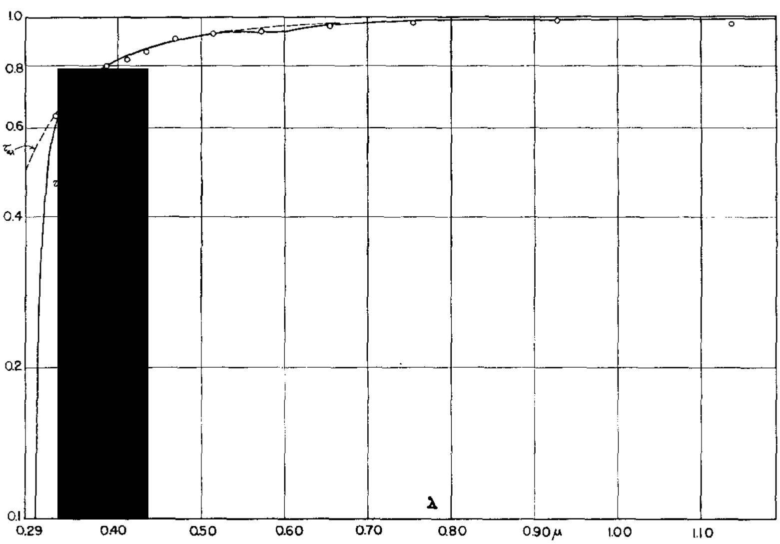
Atmospheric transmission factor for zenith sun.
○ Experimental, Tucson, Arizona (Pettit)
——— Calculated for p = 700 mm., w = 10 mm., d = 0, ozone = 1.8 mm.

various parts of the world, have been checked in a similar manner; but the foregoing examples are sufficient to show the adequacy of the method.

7. ATMOSPHERIC ABSORPTION.

The principal absorption of solar radiant energy in the earth's atmosphere occurs in the ozone bands at short wave-

FIG. 7.



Atmospheric transmission factor for zenith sun.
O Experimental, Mount Whitney (Abbot)
——— Calculated for p = 440 mm., w = 0, d = 0, ozone = 1.8 mm.

lengths and in the water-vapor bands in the infrared. Ozone absorption was considered in Section 5, while the present section will treat of water-vapor absorption, which was disregarded in the transmission curves of Section 6.

Fowle's data [23] on infrared absorption bands are plotted in Fig. 8. Beyond 2.3 μ, there is such strong absorption [24] by H_2O and CO_2 that hardly any solar radiation is transmitted.

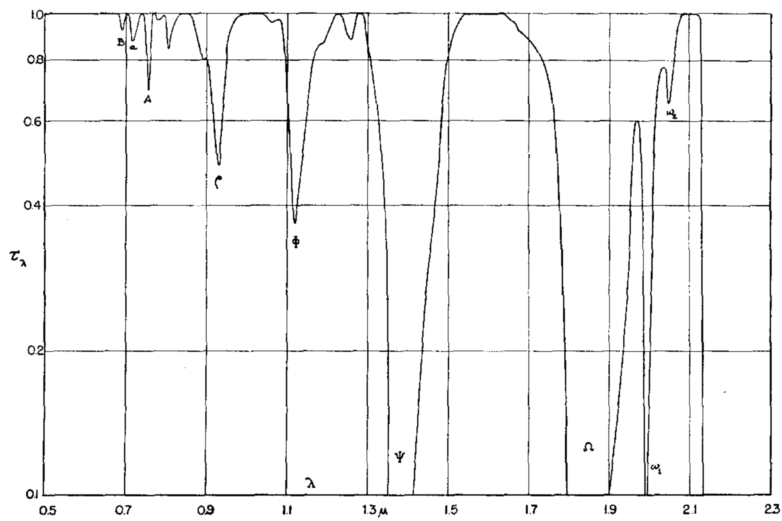
[23] F. E. Fowle, "The Transparency of Aqueous Vapor," *Astrop. J.*, **42**, 1915, Fig. 2, p. 397.

[24] F. E. Fowle, "Water-vapor Transparency to Low-temperature Radiation," Smithsonian Misc. Coll., **68**, No. 8, 1917, Fig. 7, p. 23.

I shall assume that the transmitted energy at wave-lengths beyond 2.3 μ is negligible in comparison with that at shorter wave-lengths.

The results on absorption are less satisfactory than those on scattering for two reasons. In the first place, the shape of the curve depends to some extent upon the instrument used in the measurements, particularly on the slit width of the monochromator. As the resolution is increased, the depressions in the curve become more numerous, narrower, and

FIG. 8.



Transmission factor associated with absorption by water vapor (Fowle).

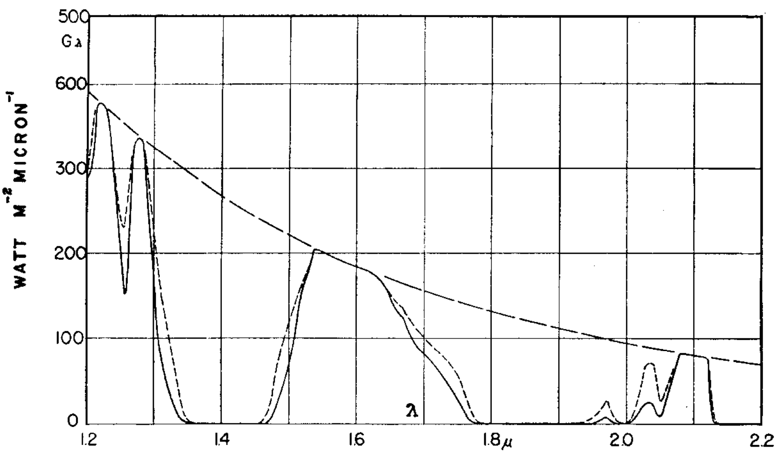
deeper.[25] This leads to the second difficulty: Bouguer's formula does not apply exactly to the data obtained with the wide slits used in spectroradiometric practice.[23]

Fowle obtained an empirical relation [23] between the apparent transmission factor at the bottom of the ρ-band and the depth of precipitable water in the atmosphere. There is still no information on how the *shape* of the transmission curve for this band varies with the amount of water vapor. I have assumed, however, that Fowle's empirical relation holds for all

[25] Abbot and Freeman, "Absorption Lines of the Infrared Solar Spectrum," Smithsonian Misc. Coll., 82, No. 1, 1929.

points in all the water-vapor bands and have obtained on this basis the upper curve of Fig. 9. The lower curve is obtained on the assumption that the Bouguer relation holds. Both curves are for sea level with 20 mm. of precipitable water directly above the station and with an air mass of 5, which is the worst condition considered in this paper. It is likely that the actual results would lie between these two extremes. In view of the small difference in areas resulting from the two assumptions, it seems advisable to use the simple Bouguer

FIG. 9.



Calculated irradiation for $m = 5$.

- - - - Fowle's method
——— Bouguer formula

relation rather than the more complicated empirical method of Fowle. Thus the Bouguer formula will be used throughout this paper in obtaining the effect of changes in water vapor, ozone, etc.

8. FINAL IRRADIATION CURVES.

Data and methods are now ready for the calculation of solar irradiation curves for any location and for any amount of water vapor, ozone, and dust in the atmosphere. The most useful results would be for a station near sea level. Accordingly, curves have been calculated for the following condition:

$$\begin{aligned} p &= 760 \text{ mm.}, \\ w &= 20 \text{ mm.}, \\ d &= 300 \text{ particles/cm.}^3, \\ \text{Ozone} &= 2.8 \text{ mm.}, \\ m &= 1, 2, 3, 4, \text{ and } 5. \end{aligned}$$

FIG. 10.

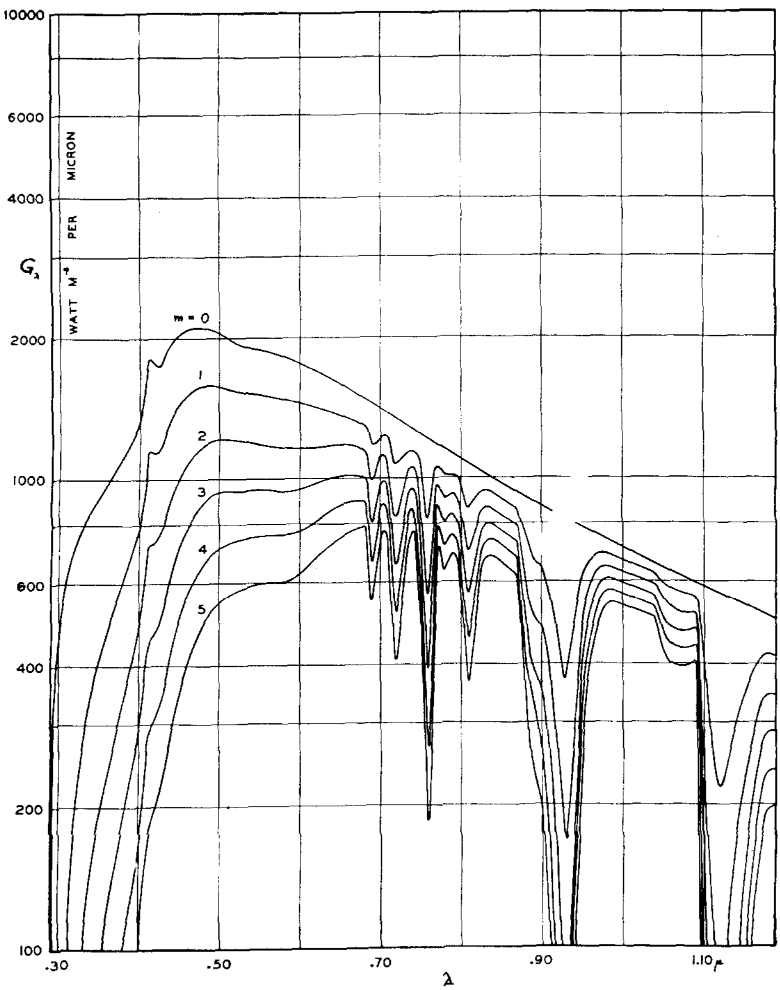
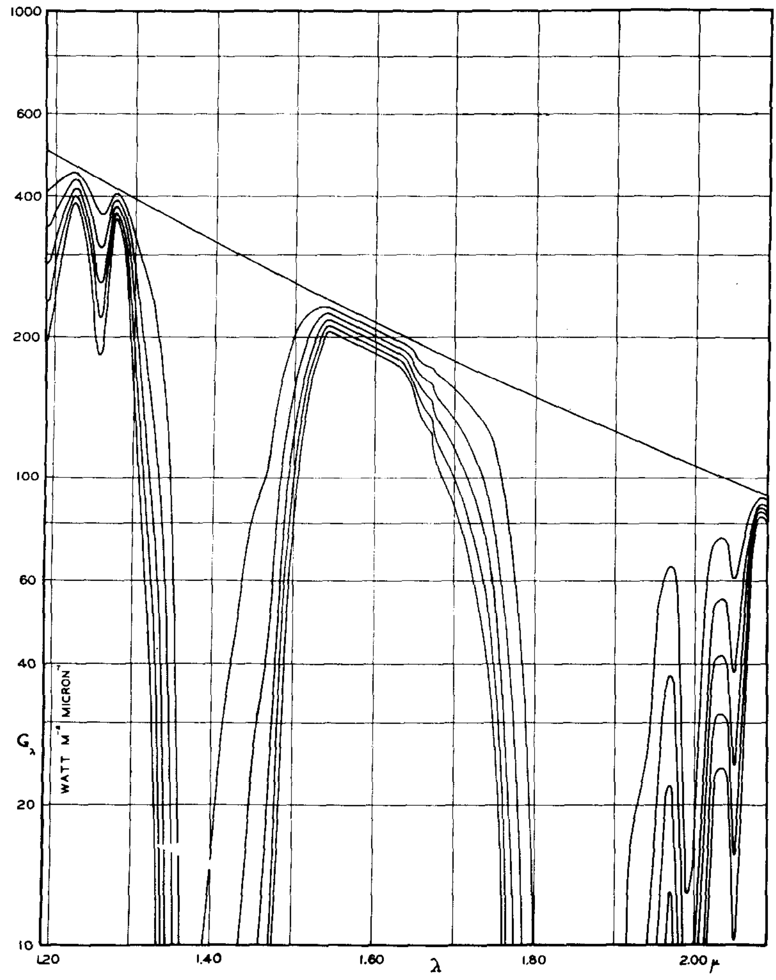


FIG. 10—*Continued.*



Solar irradiation at sea level, proposed standard curves.

p = 760 mm.
w = 20 mm.
d = 300
ozone = 2.8 mm.

This condition differs from that of Fig. 4 in the amounts of dust and ozone, which have been reduced to what appears to be more characteristic of the United States and Europe. The Smithsonian data of Fig. 4 are distinctly too low to check

TABLE III.

Solar Irradiation at Sea Level with Surface Perpendicular to Sun's Rays, $m = 2$.
(Watts per square meter per micron).

λ (microns)	G_λ	λ (microns)	G_λ	λ (microns)	G_λ
0.295	2.09×10^{-4}	0.50	1215	0.90	480
0.296	8.35	0.51	1206	0.91	375
0.297	2.87×10^{-3}	0.52	1199	0.92	258
0.298	9.87	0.53	1188	0.93	169
0.299	0.0346	0.54	1198	0.94	278
0.300	0.0810	0.55	1190	0.95	487
0.301	0.177	0.56	1182	0.96	584
0.302	0.342	0.57	1178	0.97	633
0.303	0.647	0.58	1168	0.98	645
0.304	1.16	0.59	1161	0.99	643
0.305	1.91	0.60	1167	1.00	630
0.306	2.89	0.61	1168	1.01	620
0.307	4.15	0.62	1165	1.02	610
0.308	6.11	0.63	1176	1.03	601
0.309	8.38	0.64	1175	1.04	592
0.310	11.0	0.65	1173	1.05	551
0.311	13.9	0.66	1166	1.06	526
0.312	17.2	0.67	1160	1.07	519
0.313	21.0	0.68	1149	1.08	512
0.314	25.4	0.69	978	1.09	514
0.315	30.0	0.70	1108	1.10	252
0.316	34.8	0.71	1070	1.11	126
0.317	39.8	0.72	832	1.12	69.9
0.318	44.9	0.73	965	1.13	98.3
0.319	49.5	0.74	1041	1.14	164
0.32	54.0	0.75	867	1.15	216
0.33	101	0.76	566	1.16	271
0.34	151	0.77	968	1.17	328
		0.78	907	1.18	346
0.35	188	0.79	923	1.19	344
0.36	233				
0.37	279	0.80	857	1.20	373
0.38	336	0.81	698	1.21	402
0.39	397	0.82	801	1.22	431
		0.83	863	1.23	420
0.40	470	0.84	858	1.24	387
0.41	672				
0.42	733	0.85	839	1.25	328
0.43	787	0.86	813	1.26	311
0.44	911	0.87	798	1.27	381
		0.88	614	1.28	382
0.45	1006	0.89	517	1.29	346
0.46	1080				
0.47	1138				
0.48	1183				
0.49	1210				

TABLE III.—*Continued.*

λ (microns)	G_λ	λ (microns)	G_λ	λ (microns)	G_λ
1.30	264	1.60	202	1.90	—
1.31	208	1.61	198	1.91	0.705
1.32	168	1.62	194	1.92	2.34
1.33	115	1.63	189	1.93	3.68
1.34	58.1	1.64	184	1.94	5.30
1.35	18.1	1.65	173	1.95	17.7
1.36	0.660	1.66	163	1.96	31.7
1.37	—	1.67	159	1.97	37.7
1.38	—	1.68	145	1.98	22.6
1.39	—	1.69	139	1.99	1.58
1.40	—	1.70	132	2.00	2.66
1.41	1.91	1.71	124	2.01	19.5
1.42	3.72	1.72	115	2.02	47.6
1.43	7.53	1.73	105	2.03	55.4
1.44	13.7	1.74	97.1	2.04	54.7
1.45	23.8	1.75	80.2	2.05	38.3
1.46	30.5	1.76	58.9	2.06	56.2
1.47	45.1	1.77	38.8	2.07	77.0
1.48	83.7	1.78	18.4	2.08	88.0
1.49	128	1.79	5.70	2.09	86.8
1.50	157	1.80	0.920	2.10	85.6
1.51	187	1.81	—	2.11	84.4
1.52	209	1.82	—	2.12	83.2
1.53	217	1.83	—	2.13	20.7
1.54	226	1.84	—	2.14	—
1.55	221	1.85	—		
1.56	217	1.86	—		
1.57	213	1.87	—		
1.58	209	1.88	—		
1.59	205	1.89	—		

either the Potsdam data [26] or various results obtained at the National Bureau of Standards; but if the dust is reduced to 3/8 its previous value (which was rather arbitrarily identified with 800 particles/cm.^{3}) a reasonably good agreement is obtained.

The calculated curves are shown in Fig. 10, and the data are assembled in Table III. The results are given for five values of air mass and can thus be used in obtaining the

[26] G. Müller, "Die Extinktion des Lichtes im der Erdatmosphäre und die Energieverteilung im Sonnenspektrum," Astrop. Obs. Potsdam, *Publ.*, **22**, No. 64, 1912.

irradiation or illumination at any hour of the day except near sunrise or sunset. *It is suggested that the curve for air mass 2 be used whenever a single, standard solar-radiation curve is needed in engineering calculations for places near sea level.*

If further refinement is needed, the other curves of Fig. 10 may be employed to take into account the exact zenith angle of the sun at the particular time under consideration. The amount of ozone and dust is believed to be a good average for Europe and America, while the amount of water-vapor is characteristic of summer or spring rather than winter. In some cases, it may be advisable to calculate similar curves for more water vapor (or less); and if a large amount of dust or smoke is present, suitable allowance must be made for its effect. It is believed, however, that the curves of Fig. 10 will be sufficient for most engineering purposes.[27]

9. TOTAL IRRADIATION.

If the curves of Fig. 10 are to be useful, they must give values of total irradiation, illumination, etc., that are in agreement with direct measurements. The remaining sections of this paper will be devoted to the comparison of calculated values with data obtained by direct measurements made by independent observers. Checks will be obtained on the following:

(1) Total irradiation (watt $m.^{-2}$) on a surface perpendicular to the sun's rays.
(2) Irradiation (watt $m.^{-2}$) in the extreme ultraviolet part of the solar spectrum.
(3) Time required for minimum perceptible erythema.
(4) Illumination (lumen $m.^{-2}$) on a surface perpendicular to the sun's rays.
(5) Color of direct sunlight.

Total irradiation is obtained by evaluating the areas under the curves of Fig. 10. The results are assembled in Table IV.

[27] Curves of a similar nature have been obtained by: H. H. Kimball, *C. I. E. Proc.*, 1928, p. 501; Forsythe and Christison, *J. O. S. A.*, **20**, 1930, p. 396; F. S. Brackett, "Biological Effect of Radiation," New York, 1936, Vol. I, Chap. IV; P. Moon, "Scientific Basis of Illuminating Engineering," New York, 1936, pp. 29, 30; H. P. Gage, *I. E. S. Trans.*, **34**, 1939, p. 316.

TABLE IV.

Calculated Solar Irradiation at Sea Level.

λ	G (watt m.$^{-2}$)					
	m = 0	1	2	3	4	5
0.29–0.40μ	94.6	40.1	19.8	10.0	5.4	2.7
0.40–0.70	540.0	419.7	327.8	258.6	205.8	163.7
0.70–1.1	365.4	309.2	267.5	233.4	205.1	181.5
1.1 –1.5	162.5	95.3	70.4	57.0	48.1	40.7
1.5 –1.9	72.8	50.8	45.1	41.0	38.0	35.2
1.9 – ∞	86.8	12.8	9.2	7.5	6.5	5.8
Total	1322	927.9	739.8	607.5	508.9	429.6

When plotted on logarithmic paper, the curve is concave upward (Fig. 11) as has been found experimentally by many investigators. The curve of Fig. 11 was plotted from the calculated data of Table IV. The points are experimental values for months in which the amount of water vapor is approximately the value [28] used in Fig. 10. These experimental results [29] are monthly averages for Washington, D. C., for the years 1914 to 1936, and have been reduced to mean solar distance. A fairly good agreement is noted, except that the experimental results indicate slightly greater curvature than is obtained in the calculated curve.

10. ULTRAVIOLET IRRADIATION.

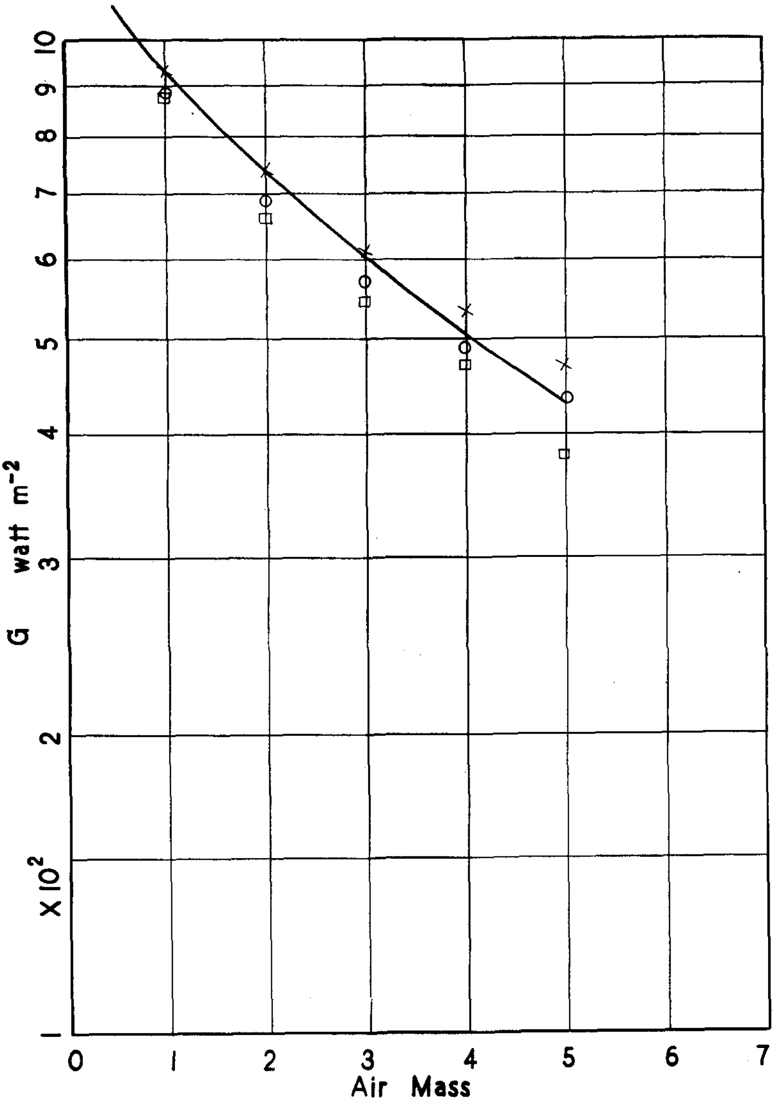
The spectral irradiation curves in the ultraviolet are replotted in Fig. 12. The conditions are the same as for Fig. 10; namely, sea level with 20 mm. of precipitable water, 300 dust particles per cubic cm., and 2.8 mm. (NTP) ozone. An outstanding feature is the very rapid decrease in ultraviolet as the air mass increases. For wave-lengths less than approximately 0.309 μ the greater part of this decrease is produced by ozone; while at longer wave-lengths, scattering by atmospheric molecules and dust particles becomes the preponderant factor.

Figure 13 shows similar irradiation curves calculated for an elevation of 2220 meters. The irradiation is greater than in

[28] H. H. Kimball, "Amount of Solar Radiation that Reaches the Surface of the Earth," *Mo. Weather Rev.*, 58, 1930, p. 51.

[29] *Mo. Weather Rev.*, 65, 1937, p. 438.

FIG. 11.



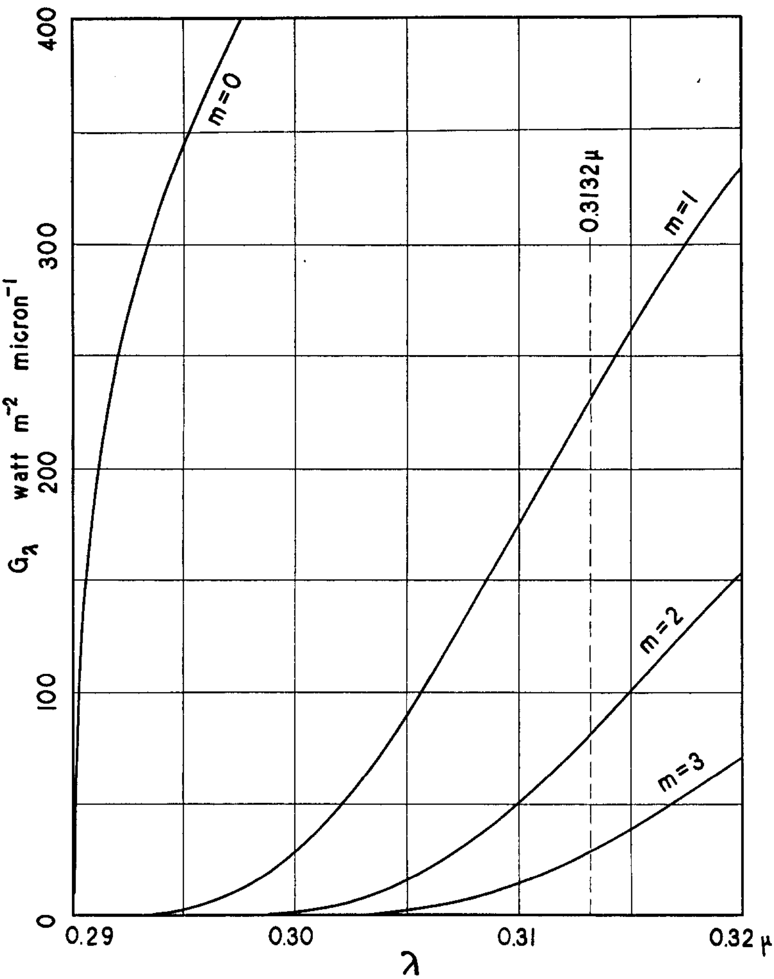
Total irradiation (watt m.$^{-2}$).

Experimental values { O May, X April, □ June

Calculated from Fig. 10 ———

the previous case and extends to shorter wave-lengths. These curves are supposed to represent conditions at Flagstaff,

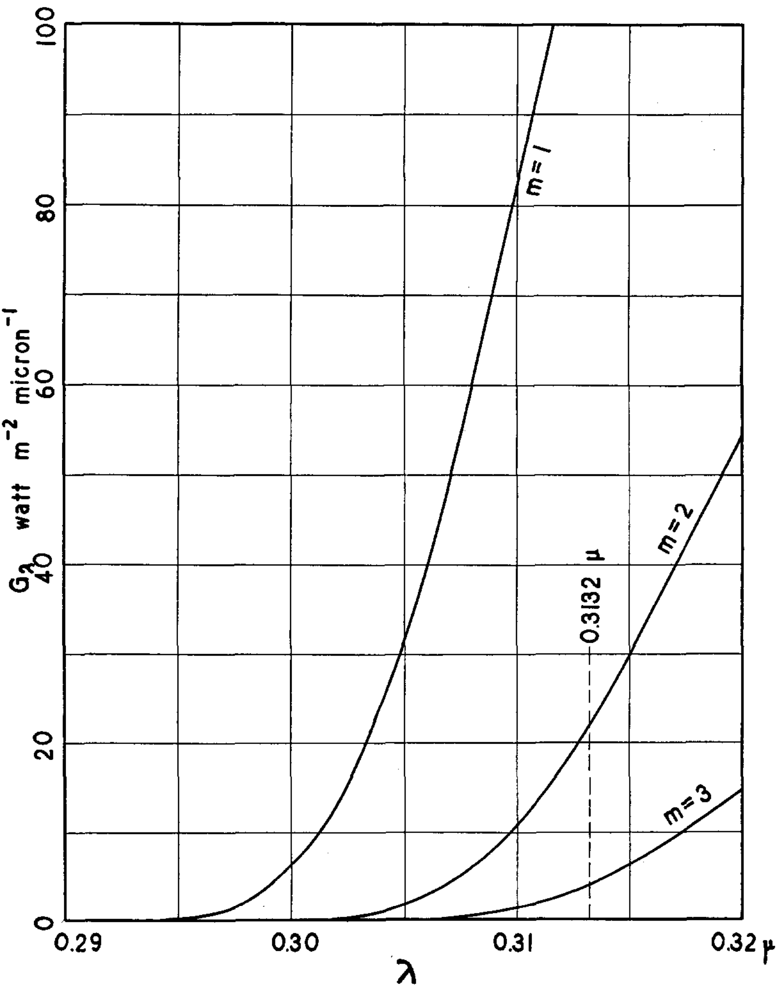
FIG. 12.



Ultraviolet irradiation, computed for conditions of Fig. 10.

Arizona, and are calculated for a pressure of 580 mm., zero water vapor and zero dust. A value of 1.8 mm. of ozone is used in the computations, since this amount is in agreement with the measurements of Pettit [9] at Tucson, Arizona, and of Fowle at Harqua Halla.

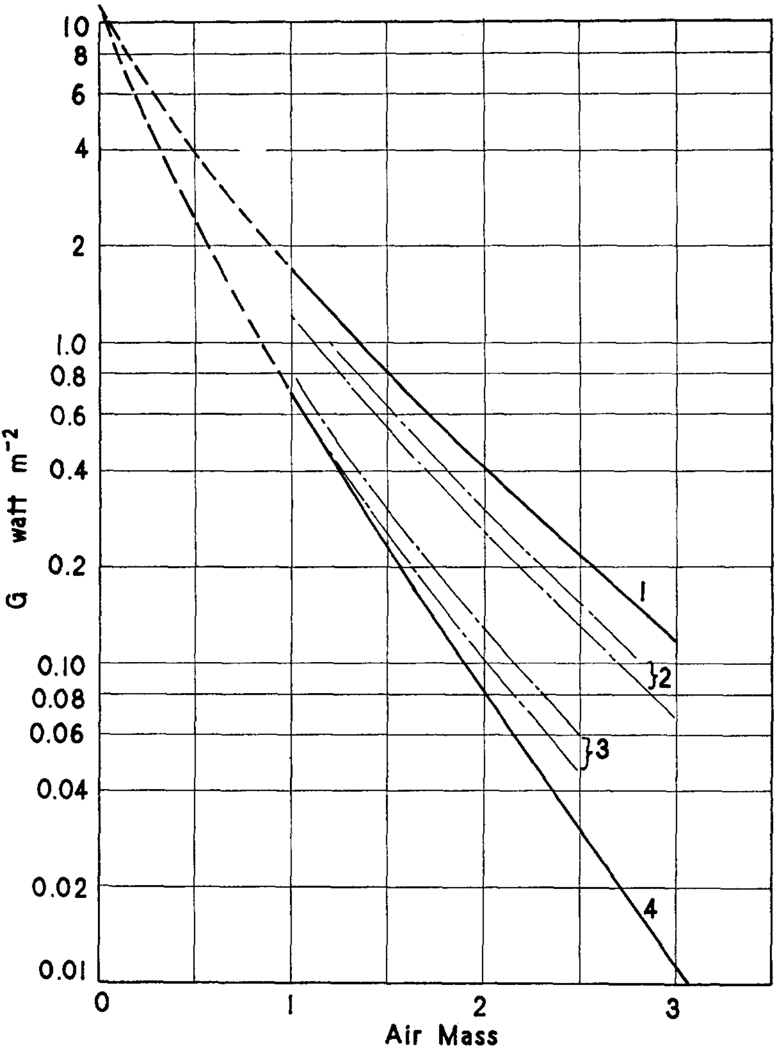
FIG. 13.



Ultraviolet irradiation, computed for p = 580 mm., w = 0, d = 0, ozone = 1.8 mm.

The total irradiation (watts/m.^{2}) for wave-lengths less than 0.3132 μ was obtained by integration of the curves of Figs. 12 and 13. The values are shown by the circles in Fig. 14. Curves 2 and 3 were obtained experimentally by Coblentz

FIG. 14.



Total irradiation at wave-lengths less than 0.3132μ.

1. Calculated for p = 580 mm.
2. Experimental values for Flagstaff, Ariz. (Coblentz).
3. Experimental values for Washington, D. C. (Coblentz).
4. Calculated for p = 760 mm.

and Stair[30] at Flagstaff and Washington respectively. The calculated results for Flagstaff are approximately 20 per cent. higher than the experimental results. This discrepancy could be reduced by assuming an appreciable amount of dust and water vapor in the atmosphere, but it is more likely caused by a slightly greater amount of ozone than the assumed value of 1.8 mm. The calculated results for Washington are considerably lower than the experimental values, though almost perfect agreement could be obtained by a slight change in the assumed amount of ozone. The shape of the calculated and experimental curves is gratifyingly similar; and though the numerical check is not very satisfactory, it is probably as good as can be expected at the present state of the art.

Because of the very narrow band of wave-lengths under consideration, one might guess that the curves of Fig. 14 would be straight lines, as they would be for truly homogeneous radiation. In fact, this assumption is made by Coblentz,[31] who extends the line to zero air mass and obtains in this manner an irradiation of 6.0 watt/m.^{2} outside the atmosphere. Actually, however, the irradiation changes very rapidly in this narrow band of wave-lengths so it is anything but homogeneous. As a result, the curves are distinctly concave upward and the calculated irradiation outside the atmosphere is 10.62 watts/m.^{2} between 0.2900 and 0.3132 μ. It seems likely that this result is more nearly correct than the value of 6.0 given by Coblentz.

It is interesting also to calculate the time required for minimum perceptible erythema at Washington and at Flagstaff. The curves of Figs. 12 and 13 for $m = 1$ were evaluated according to relative erythemal effectiveness,[32] and the integrals were obtained, giving

Washington, 0.140 erythemalwatts/m.^{2}, $t = 29.8$ minutes,
Flagstaff, 0.410 erythemalwatts/m.^{2}, $t = 10.1$ minutes.

The times (t) for MPE were obtained by using the Luckiesh

[30] Coblentz and Stair, "Evaluation of Ultraviolet Solar Radiation of Short Wave-lengths," *B. S. J. R.*, 16, 1936, Fig. 7 (p. 337) and Fig. 11 (p. 343).

[31] Ibid., p. 345.

[32] *C. I. E. Compte Rendu*, 1935, p. 625.

value [33] of 250 erythemalwatt-sec./m.^{2} for MPE. These calculations are in approximate agreement with Coblentz's experience [34] that MPE was obtained in Washington at noon in midsummer in 20 to 30 minutes, while at Flagstaff only 9 or 10 minutes were required.

11. ILLUMINATION AND COLOR.

The proposed standard curves of Table III can be used also in the calculation of direct solar illumination. The data were multiplied by the standard visibility function v, and integrals of the form

$$E = \int_0^\infty vG_\lambda d\lambda \qquad \text{(lightwatt m.}^{-2}\text{)}$$

were evaluated. A conversion factor of 621 lumens = 1 lightwatt then gave the illumination values of Table V. These are illuminations for direct sunlight only (no skylight) and are for a surface perpendicular to the sun's rays.

Very few experimental data are available as a check on the calculated results of Table V, since most illumination measurements include the effect of skylight. However, Kimball gives a set of conversion factors [29] which he obtained at Washington by the use of an illuminometer of the visual-comparison type. These factors give the relation between direct solar illumi-

TABLE V.

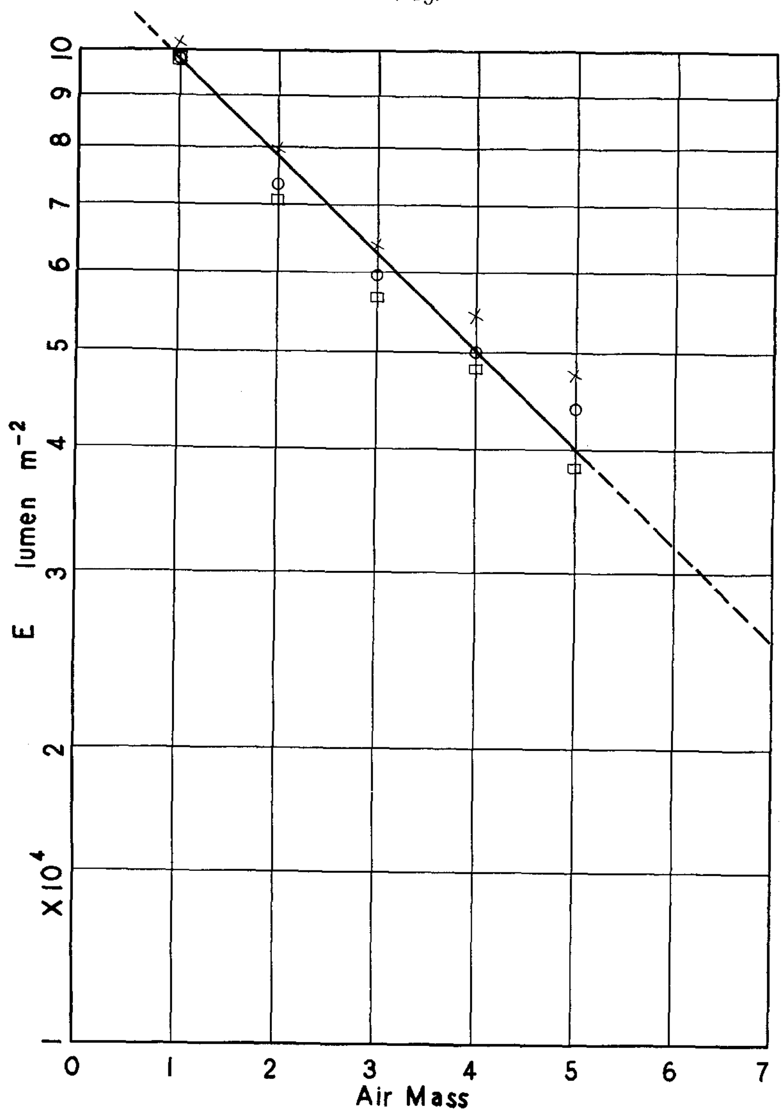
Calculated Illumination and Color of Direct Sunlight at Sea Level.

	$m = 0$	1	2	3	4	5
E (lumen m.$^{-2}$)	123,400	98,300	78,400	62,800	50,100	40,000
x	0.3179	0.3306	0.3431	0.3558	0.3676	0.3803
y	0.3297	0.3437	0.3567	0.3687	0.3789	0.3881
z	0.3524	0.3257	0.3002	0.2755	0.2535	0.2316
$1/Tc \times 10^{+6}$	161.2	180.0	196.6	214.0	230.3	246.8
Tc (°K)	6,200	5,550	5,090	4,670	4,340	4,050

[33] Luckiesh and Holladay, "Fundamental Units and Terms for Biologically-effective Radiation," *J. O. S. A.*, **23**, 1933, p. 197.

[34] Coblentz and Stair, "Factors Affecting Ultraviolet Solar-Radiation Intensities," *B. S. J. R.*, **15**, 1935, p. 142.

FIG. 15.



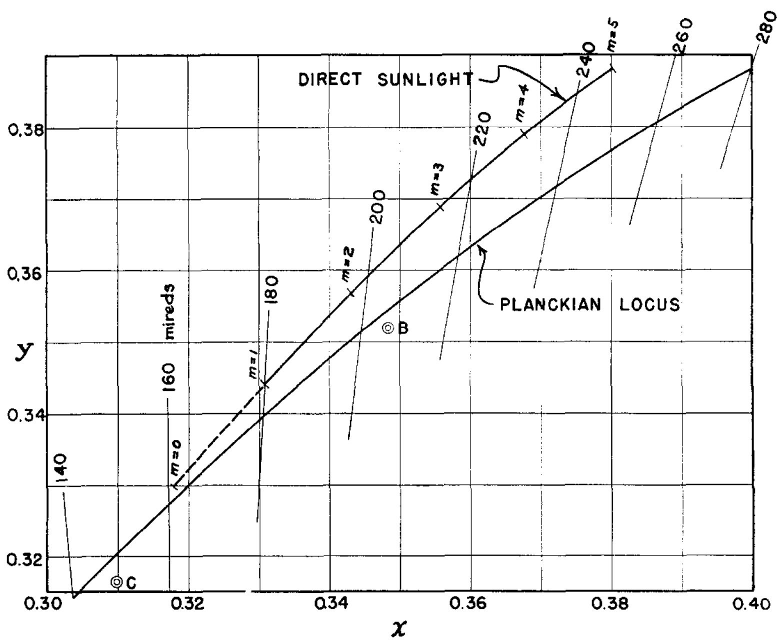
Direct solar illumination (lumen m.$^{-2}$).

Experimental values { ○ May, × April, □ June }

Calculated from Fig. 10 ———

nation and total irradiation for various values of m. Multiplication of the previous experimental values of G (Fig. 11) by the Kimball illumination factors gives the points of Fig. 15. The curve is the calculated one, plotted from the data of Table V. It appears to be a perfectly straight line; which does not seem unreasonable, in view of the uniformity of the G_λ-curves in the visible region. The experimental points

FIG. 16.

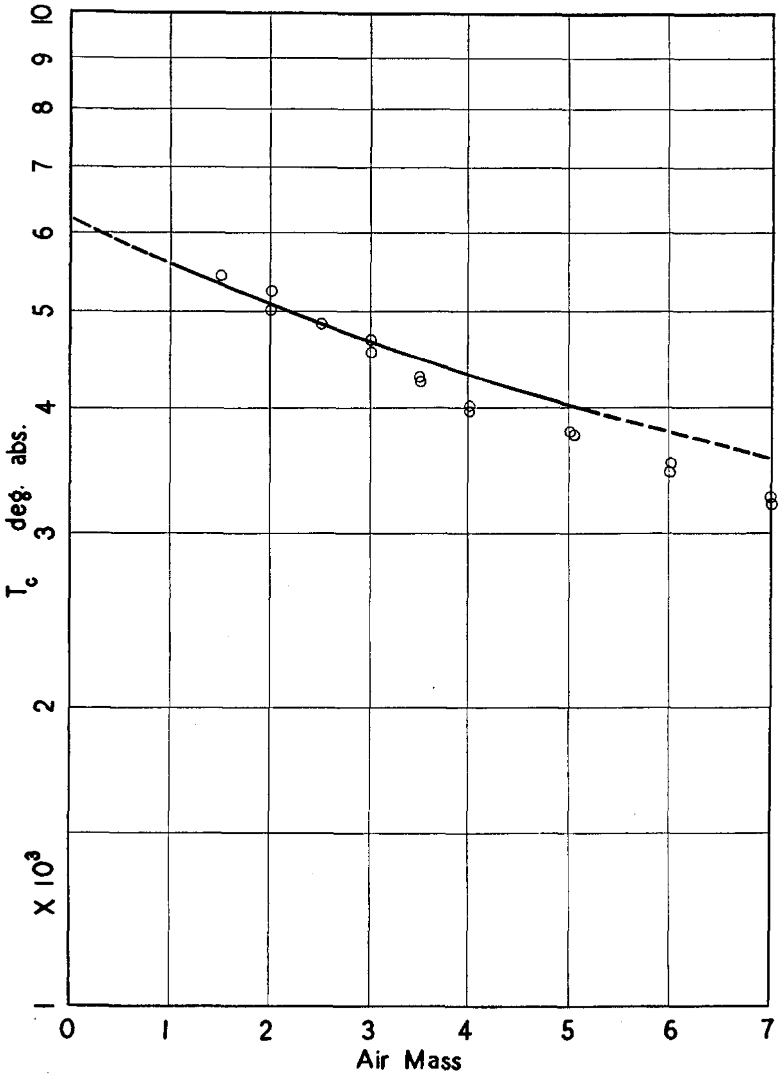


Color of direct sunlight, calculated for sea level by use of Fig. 10.

show a distinct curvature but are otherwise in approximate agreement with the calculated curve.

Table V contains also the coördinates in the color triangle, calculated by use of the C.I.E. standard data. The calculated locus for direct sunlight at sea level is shown in Fig. 16. For sunlight outside the atmosphere, the proposed standard of Table I gives a point ($m = 0$) very close to a Planckian distribution wi.h temperature 6200° K. As the air mass increases, the sunlight becomes progressively further above the Planckian

FIG. 17.



Color temperature of direct sunlight at Washington.
Experimental data of Priest ○
Calculated from Fig. 10 ———

locus, which may explain Priest's difficulty[35] in obtaining a color match between a Planckian distribution and sunlight with low sun.

To obtain the closest color temperature T_c for the six calculated points, I have used the method and data of Judd,[36] as indicated by the lines of equal $(1/Tc)$ drawn in Fig. 16. The curve of Fig. 17 was plotted from the resulting values of T_c. Priest's data, obtained at Washington in 1920 by use of the Leucoscope, are shown by the points.[37] These data are not averages, but were obtained during a single day. It is likely that measurements for other days, had they been obtained, would have differed by amounts at least as large as those between the points and curve of Fig. 17.

12. SUMMARY.

A wealth of data on sunlight has been obtained by astrophysicists, meteorologists, and others; but this information has been scattered through the literature and has not been generally available to engineers. The present paper correlates some of the data and specifies a proposed standard spectral-distribution curve for sunlight outside the atmosphere. Methods are given also for the calculation of the spectral irradiation curve for any elevation above sea level and for any air mass, and these methods lead to proposed standard curves to be used in engineering calculations dealing with direct sunlight at sea level.

These curves are checked against independent data on total irradiation, ultraviolet irradiation, illumination, and color temperature. In all cases, the agreement between calculated and experimental results gives confidence in the validity of the proposed curves for various engineering applications.

[35] I. G. Priest, "Preliminary Data on the Color of Daylight at Washington," *J. O. S. A.*, 7, 1923, p. 78.

[36] D. B. Judd, "Estimation of Chromaticity Differences and Nearest Color Temperature on the Standard 1931 I.C.I. Colorimetric Coördinate System," *J. O. S. A.*, 26, 1936, p. 421.

[37] As given by Kimball, *Mo. Weather Rev.*, 52, 1924, Fig. 2, p. 475.